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Brief Communication

The study of free rise of buoyant spheres in gas reveals the universal behaviour of free rising rigid spheres in fluid in general

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1. Introduction

Since ancient times people have been trying to understand the nature of the forces governing the free fall of different objects in gas or liquid. Closely related to it is another type of free motion of bodies in a fluid, usually liquid: the free rise of buoyant fluid or rigid particles such as gas bubbles, ice crystals, wood particles, cork, etc. Galileo Galilei was the first who identified the driving force (the particle–fluid density difference) of both free fall and free rise (Galilei, 1590), as well as the force acting against it (i.e., friction) (Galilei, 1638). On the basis of the balance of these forces, Galileo was the first researcher who assumed that a free rising buoyant sphere (for example, inflated bladder rising in water) would move exactly the same way as a free falling heavy sphere (stone in water). Since the publication of Galileo's works, it has been universally assumed, explicitly or not, that free fall and free rise of rigid particles should have similar motion properties (Newton, 1760; Knudsen and Katz, 1958; Munson et al., 1990; White, 1999).

The above-mentioned assumption was based on the balance of forces applied to a freely settling or rising sphere. The difference between the gravity and Archimedes forces (the driving force of the process) depends on the particle volume and the particle–fluid density difference, while the drag force is a function of the fluid properties such as density (ρ_f); particle velocity (U_t) and particle diameter (d_p). In addition, the drag coefficient C_D, defined as $C_D = (4\ \mathrm{gdp}|\rho_f - \rho_p|)/(3\rho_f U_t^2)$ is a function of the Reynolds number, defined as $Re = U_t d_p \rho_f/\mu$. At steady-state conditions, the driving force is equal to the drag force. Therefore, if a buoyant and a heavy particle with the same

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diameter and the same absolute value of fluid–particle density difference are placed in the same fluid, they were supposed to have the same terminal velocity, trajectory and a drag coefficient. The only difference should be in the direction of the motion: upwards and downwards, respectively.

The relationship between the drag coefficient and Reynolds number for the case of free falling spherical particles in quiescent fluid as well as for immobilized spheres in a moving fluid is represented by the well-known standard drag curve. The best correlation describing it (Karamanev, 1996) is the one proposed by Turton and Levenspiel, 1986

$$C_{\mathrm{D}} = \frac{24}{Re}\left(1 + 0.173Re^{0.657}\right) + \frac{0.413}{1 + 16300Re^{-1.09}}. \tag{1}$$

It took more than 300 years after the publication of Galileo's works until another study on free rising particles was published. Allen (1900) studied the rise of small paraffin spheres (the diameter of only one of the particles studied exceeded 1 mm) in aniline; the terminal Reynolds numbers were below 30. His experiments confirmed Galileo's theory on the similarity of free rise and fall. It has been shown that rising paraffin spheres have the same trajectories and drag coefficients as free falling spheres. It took nearly 100 more years until the first systematic study of the hydrodynamics of free rise of light spherical particles in liquid (Karamanev and Nikolov, 1992) was published. In that paper we showed that rising light spheres in a Newtonian liquid (water) exhibited a completely different behaviour from that of free falling spheres in the same liquid, especially at higher Reynolds numbers. It has been shown that free rising in water light spheres follows a spiral path. The angle between the spiral tangent (the velocity vector) and the horizontal plane of all the spirals, regardless of the particle size and terminal velocity, was almost constant and fluctuated around 61°. The drag coefficients of these spheres were equal to those for free falling ones (corresponding to the standard drag curve, Eq. (4)) only when the Reynolds number was below 135 and/or when the sphere density was close to that of the surrounding fluid. At higher Reynolds numbers, the drag coefficient was a constant, equal to 0.95. This value was more than two times larger than the drag coefficient in Newton's law region. Similar results were obtained with the rise of rigid spheres in non-Newtonian (pseudoplastic) liquids (Dewsbury et al., 2000).

However, it is not yet known if this type of behaviour can be observed in the case of spheres rising in gas. Published experimental data show that there is a significant horizontal velocity component when spherical meteorological balloons, with diameters between 0.5 and 2 m, rise in a calm atmosphere (Scoggins, 1965). However, most experiments with balloons have been performed at supercritical conditions ($Re > 3 \times 10^5$) when the horizontal motion was highly erratic. On the other hand, more organized, helical motion was observed at Reynolds numbers just below the critical point (Rogers and Camnitz, 1966). No information is available on the rise of low-density spheres in gas at Reynolds numbers below approx. 10^4–10^5. Taking into account the results concerning the rise of light spheres in liquid, it is of great interest to study the free rise of spheres with a much lower density than that of the surrounding gas.

The main goal of this work is to study the dynamics of the rise of low-density rigid (or rather, semi-rigid) spheres in gas and to compare their behaviour to that of rigid spheres rising in liquid.

2. Materials and methods

In order to study the dynamics of the free rise of spheres with density much lower than that of the surrounding gas, especially with size in the order of centimetres, one of the greatest challenges is to produce such spheres. The spheres used in this study were soap bubbles filled with helium or hydrogen. The surrounding gas was air. Because of the effect of surfactant molecules, the surface of these bubbles was immobile, and their shape remained very close to a spherical one. The aspect ratio was always above 0.9.

In order to produce a stable and rigid film, the following chemical solution was used (Walker, 1987): 85 ml of glycerol, 25 ml deionized water, 1.4 g triethanolamine and 2.0 g oleic acid. The bubbles were produced by blowing helium or hydrogen through tubes with openings between 200 μm and 1.5 cm in diameter. In general, tubes with larger openings produced larger bubbles.

The terminal velocity of the rising bubbles was determined by stroboscopic photography with a frequency between 17 and 60 s^{-1}. The bubble size was also determined from the photographs. The photographic camera used was 35-mm Nikon LG. A GenRad 1546 Strobotac digital stroboscope was also used. The bubbles were released in a dark rectangular chamber with $L \times W \times H = 1 \times 1 \times 2.5$ m which had a front plexiglass wall. The stroboscopic light came from the top of the chamber and was directed downwards. The terminal velocity was calculated on the basis of the vertical velocity component. The air temperature was 22°C.

3. Results and discussion

The trajectories of free rising and falling, solid and fluid particles in Newtonian liquid (water) are shown in Fig. 1. At Reynolds numbers higher than 135 and low particle density, the free rising solid spheres followed a spiral path (Fig. 1(a)–(c)), as opposed to a rectilinear one (Fig. 1(e)) in the case of falling spheres in the same liquid. However, the motion of free rising spheres was similar to that of falling spheres (i.e., it was linear) at $Re < 135$ (Karamanev and Nikolov, 1992). As already mentioned, the angle between the velocity vector (the spiral tangent) and the horizontal plane is always a constant, close to 61° with a deviation of approx. 5% when buoyant rigid spheres rise in Newtonian liquids (Karamanev and Nikolov, 1992). The spherical particles rising in non-Newtonian liquids with $Re > 135$ also followed a spiral path with a mean spiral angle of 60° (Dewsbury et al., 2000), which is practically the same as that in Newtonian liquids. In addition, gas bubbles with spheroidal and ellipsoidal shape, rising in Newtonian liquids, also followed a spiral trajectory with a similar angle (Fig. 1(d)). The differences between the trajectories of falling and rising particles in liquid were explained by the effect of fluid turbulence, and in particular by the rotating vorticity, on the motion of particles with different mechanical inertia (Karamanev et al., 1996).

The results of this paper show that trajectory of the rising spheres in air was also spiral (Fig. 2). The angle between the velocity vector and the horizontal plane was a constant, equal to 64° with ±6% deviation at all sizes and Reynolds numbers studied. Only the spheres with Reynolds numbers below approx. 500 had a less regular spiral trajectory. Therefore, the angle of the velocity vector of spheres rising in gas can be considered equal, within the experimental error, to that of spheres rising in liquids.

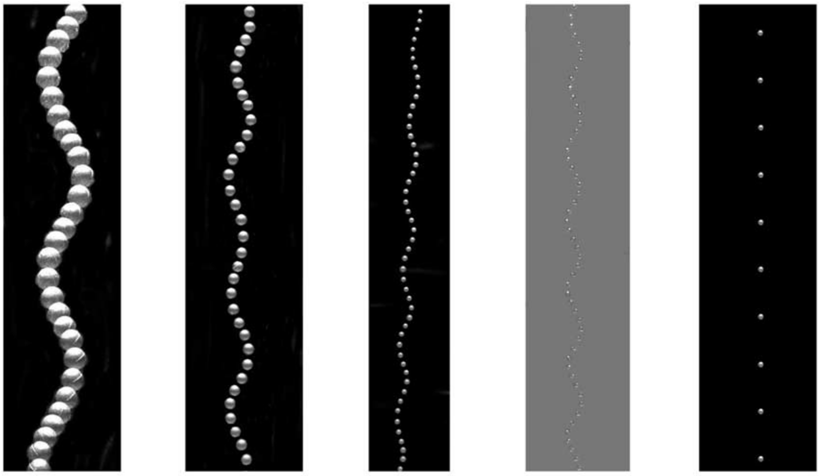

Fig. 1. Stroboscopic photographs of rising and falling solid spheres and rising air bubbles in water. (a)–(c) rising Styrofoam spheres with diameters of 12, 6.3 and 3.1 mm, respectively, and a density of 0.049 g/cm^3; (d) rising gas bubble with an equivalent diameter of 3.5 mm; (e) falling stainless steel sphere with a diameter of 4 mm and a density of 7.85 g/cm^3. The frequency of the stroboscopic light was different for each picture and was in the range between 25 and 60 s^{-1}.

One of the most important characteristics of particle motion in a fluid is the relationship between the drag coefficient (C_D) and the Reynolds number. In order to calculate the drag coefficient of the bubble rise (as defined above), one must know the density of the bubble (ρ_p). The major parameter affecting the bubble density is the soap film thickness. It has been shown in the literature that there is a stable film thickness (black film) which is approx. 300 Å (Isenberg, 1978). It was calculated that at film thicknesses below 2000 Å, the overall bubble density, and especially the density difference, were not significantly affected by the film thickness when the bubble diameter was greater than 1 cm (Fig. 3). This is because the mass of the soap film was much smaller than the mass of the gas inside the bubble. In addition, the film thickness of the bubbles studied in this work was estimated by the colour of the light reflected from the bubble film. Since the film thickness and the wavelength of visible light have similar values, the interference of light reflected from both gas–film interfaces produces a bright colour. When the film thickness is below 2000 Å, the interference becomes destructive at any angle of reflection, which results in a blackening of the film (Isenberg, 1978). All the soap bubbles used in this work were black in colour, and so their thickness was much below 2000 Å. Therefore, under these conditions the density difference between the bubble and the surrounding gas was very close to the difference between the densities of the gases inside and outside the bubble (Fig. 3).

As mentioned above, free rising solid spheres in liquid have the same drag coefficient (as well as trajectory and terminal velocity) as falling ones only when the Reynolds number was below 135. At higher Reynolds numbers, the drag coefficient of rising particles is a constant, equal to 0.95 (Karamanev and Nikolov, 1992). The C_D–Re relationship in the case of falling spheres (standard drag curve) is shown in Fig. 4 and was calculated using the correlation of Turton and Levenspiel, 1986 The drag curve of rising spheres with $\rho_p \ll \rho_f$ in Newtonian liquids is also shown in Fig. 4.

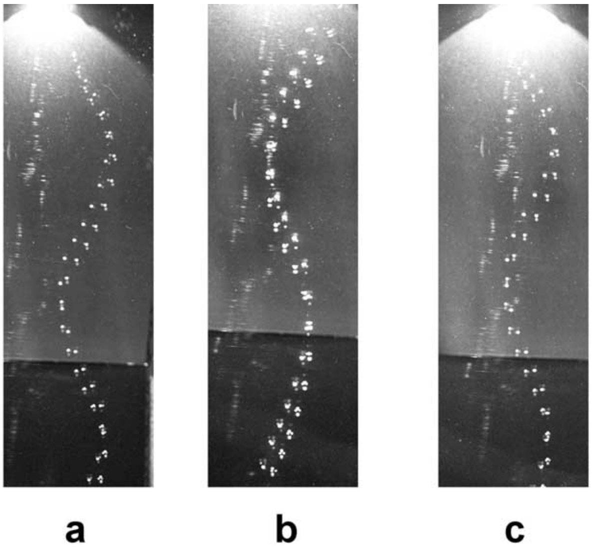

Fig. 2. Typical trajectories of rising spherical bubbles in air. Bubble diameters: (a) 5.0 cm; (b) 6.3 cm; (c) 9.8 cm.

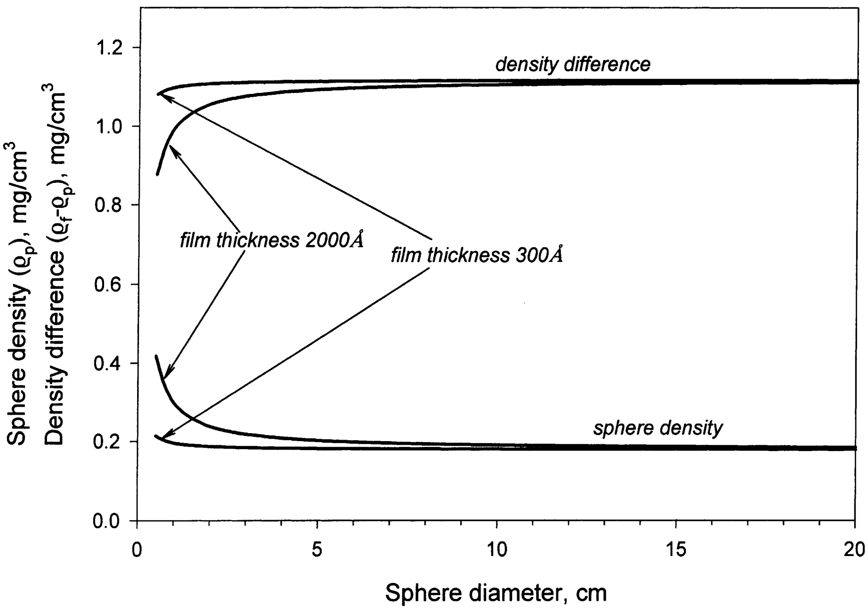

Fig. 3. The effect of bubble wall thickness on the overall bubble density and fluid–particle density difference.

The latter curve also describes the drag coefficient of rigid spheres rising in non-Newtonian fluids (Dewsbury et al., 2000), as well as that of gas bubbles in contaminated Newtonian liquids (Karamanev, 1994).

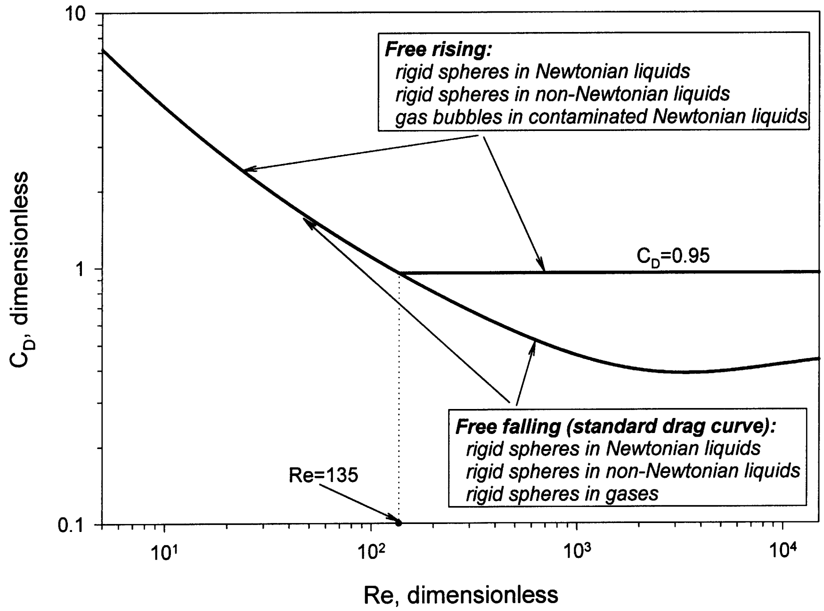

Fig. 4. The relationships between the drag coefficient and Reynolds number (drag curves) for different free moving rigid and fluid particles. Literature sources: Free rising: rigid spheres in Newtonian (Karamanev and Nikolov, 1992; Karamanev et al., 1996) and non-Newtonian (Dewsbury et al., 2000) liquids; gas bubbles in contaminated Newtonian liquids (Karamanev, 1994). The standard drag curve: Eq. (1). Free falling rigid spheres in Newtonian (Clift et al., 1978) and non-Newtonian (Chhabra, 1993) liquids and in gases (Clift et al., 1978).

The effect of the Reynolds number on the drag coefficient of spheres rising in air is shown in Fig. 5. It can be seen that the relationship is the same as that of rising spheres in liquids (Fig. 4). Unfortunately, the rise of bubbles with diameters smaller than 8 mm, corresponding to $Re < 200$ was not studied because it was impossible to determine their density precisely enough.

As a result of this study, an important conclusion was made. The rise of low-density rigid spheres in any fluid (gas, Newtonian and non-Newtonian liquid) obeys the same universal pattern. When the Reynolds number is higher than 135 and the sphere density is much smaller than that of the fluid, the motion is spiral with a spiral angle of 60–64°, and the drag coefficient is a constant, equal to 0.95. At Reynolds numbers below 135, the drag coefficient follows the standard drag curve and the trajectory is linear. The latter could not be verified for the case of the rise of spheres in gas due to the reasons given above. This behaviour of rising solid particles is similar to that of rising gas bubbles. However, the bubble surface motion leads to differences in some cases.

These findings are important for predicting the hydrodynamics of rise of such fluid and solid rising objects as gas bubbles and buoyant solid particles in liquid and rising meteorological balloons in the atmosphere. The results will be of great significance to the better understanding of different multiphase processes in chemical, biochemical, metallurgical and other industries, as well as in atmospheric and space research (i.e., the rise of meteorological balloons in the atmospheres on Earth, Mars and Venus (Nishimura et al., 1991)).

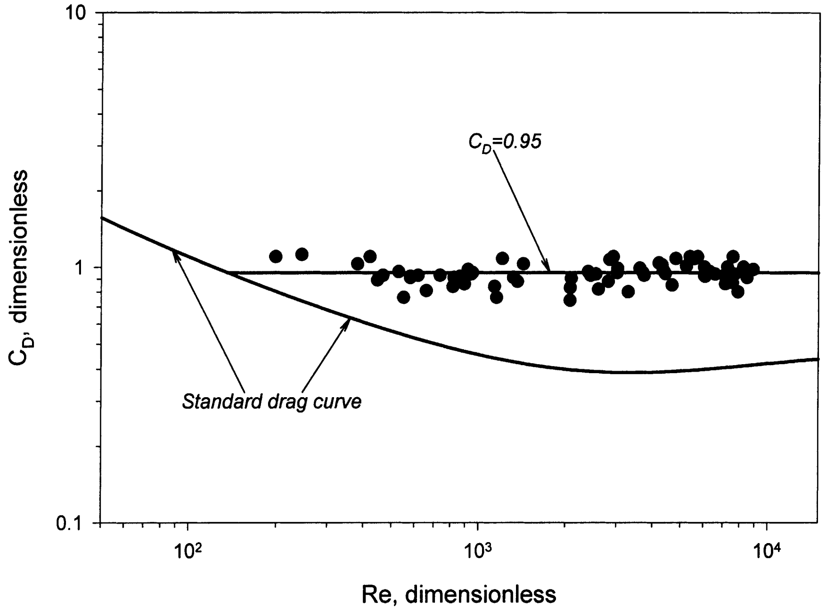

Fig. 5. The effect of Reynolds number on C_D for rising spherical soap bubbles in air.

Acknowledgements

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